$W, Z + 2$ jet production at NLO

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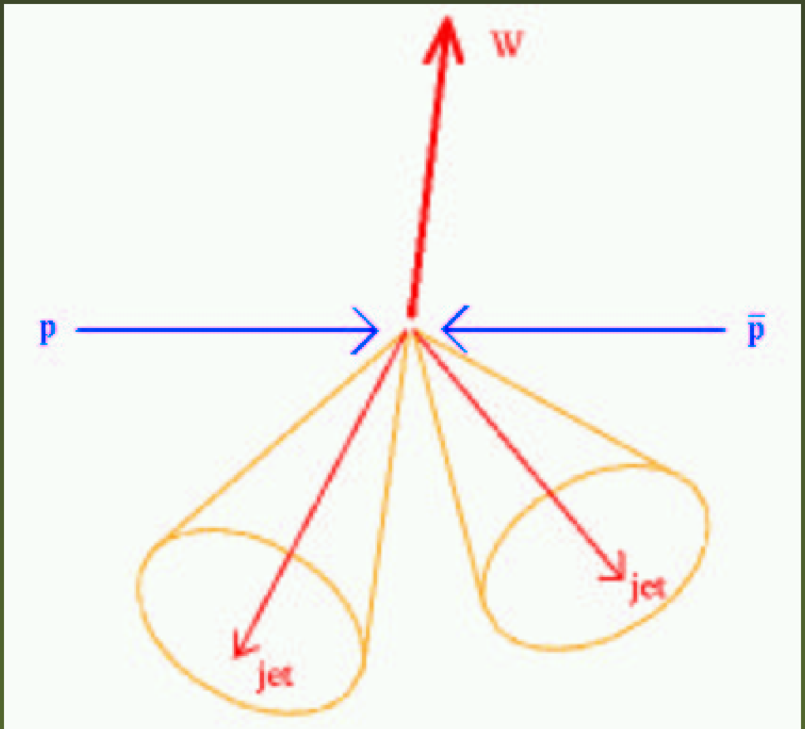
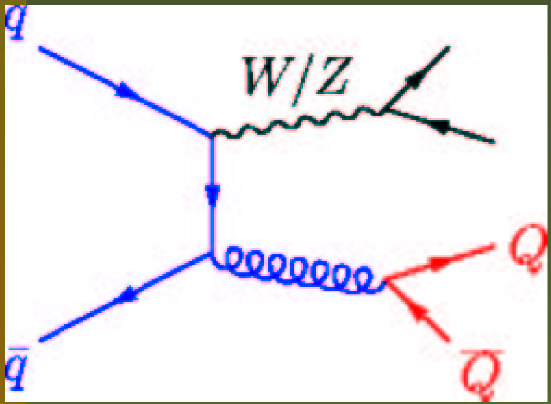
W+2 jet events

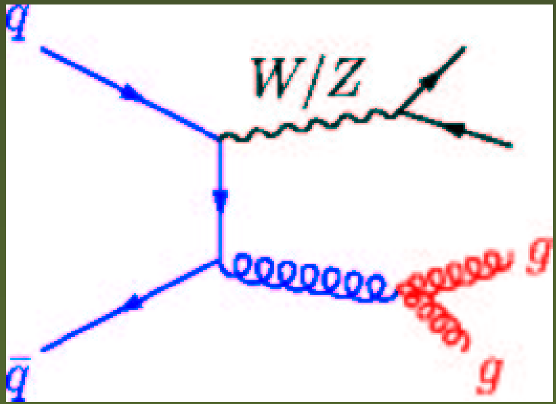
- Many such events at Run I of the Tevatron. For example, with an integrated luminosity of $108\ \mathrm{pb}^{-1}$ CDF collected 51400 $W \rightarrow e\nu$ events, of which 2000 are $W + 2$ jet events. This yields an $80\mathrm{pb}$ cross-section.



W+2 jet theory

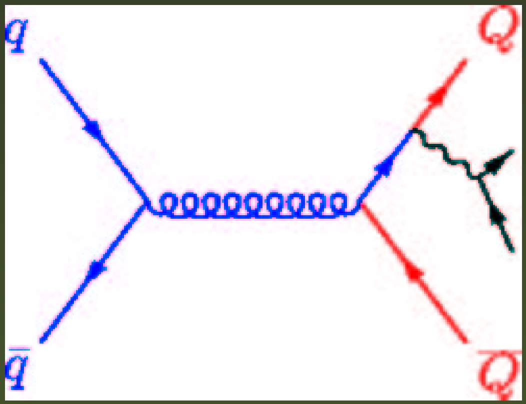
- In the leading order of perturbative QCD, this process can be represented by Feynman tree-graphs. The (anti-)proton contains quarks and gluons which provide the initial state.
- At leading order a jet is represented by a single quark or gluon in the final state. Local Parton-Hadron Duality suggests this is a good approximation.

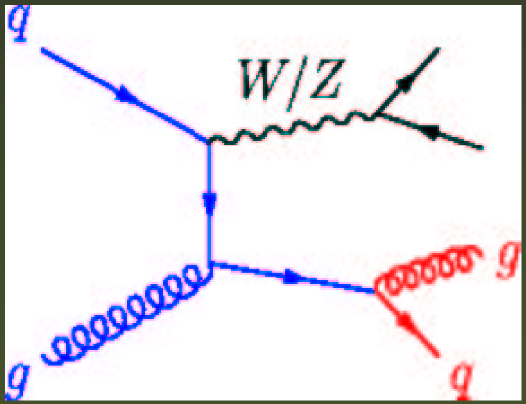


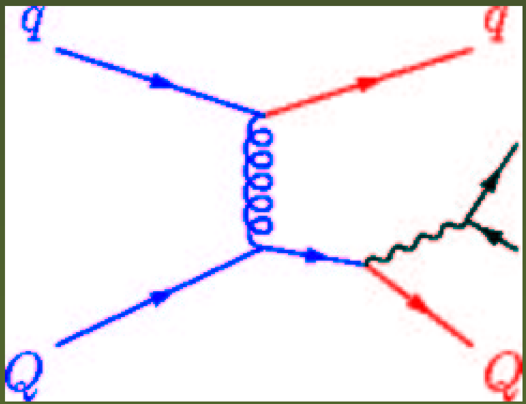


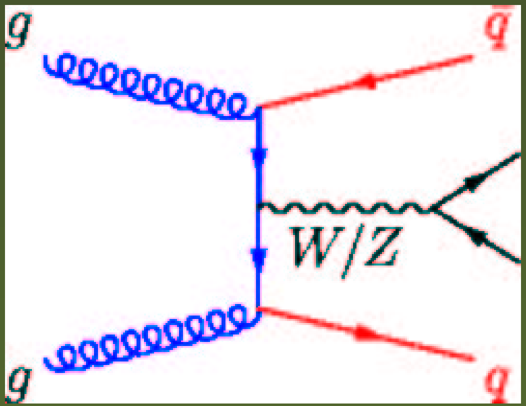
W+2 jet theory, continued

- Related diagrams provide other initial states that also contribute:



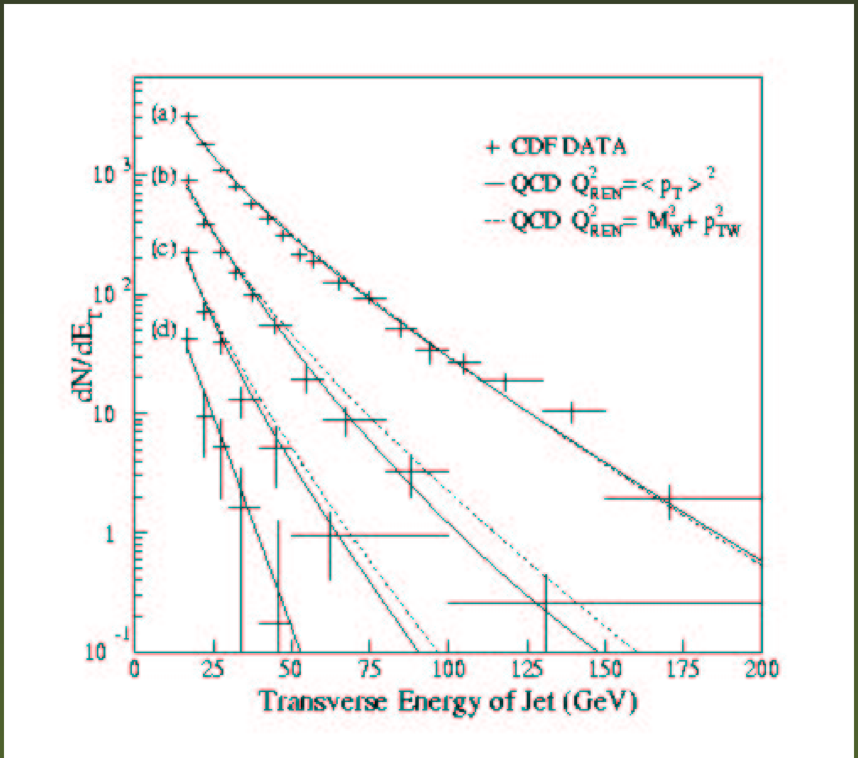






Multi-jet data

- This theory describes multi-jet data fairly well. For example, the leading-jet E_T spectrum for $W + n$ jet production ($n = 1, \ldots, 4$):



- Deficiency at high E_T in the $W + 1$ jet sample.

Failings of leading order

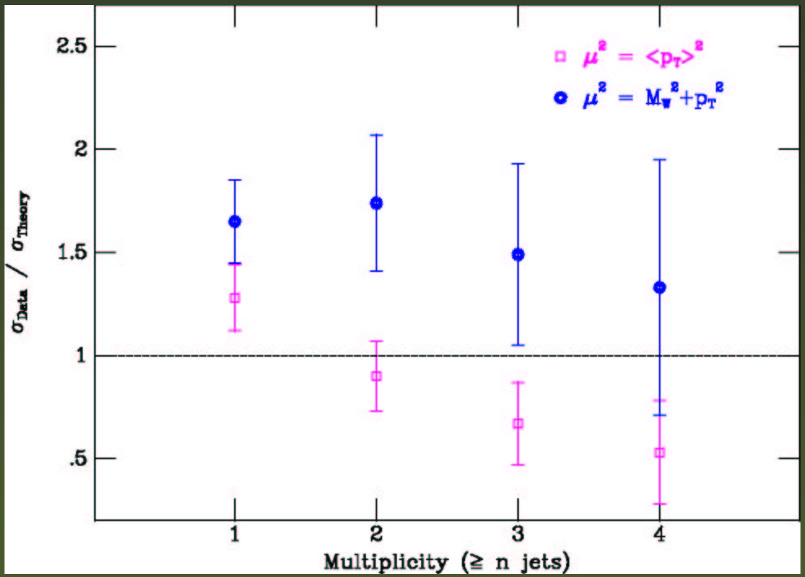
- Some discrepancies arise when the theory is examined in more detail.
- An important theoretical input is the value of the renormalization and factorization scales, μ_R and μ_F.
- These artificial variables are required only because we cannot solve the full theory of QCD. Instead, we compute scattering amplitudes perturbatively,

$$|\mathcal{M}_{\text{full}}^{2-\text{jet}}|^2 = \alpha_S^2|\mathcal{M}_2|^2 + \alpha_S^3|\mathcal{M}_3|^2 + \ldots + \alpha_S^r|\mathcal{M}_r|^2 + \ldots$$

- Truncating this series produces a dependence upon μ_R and μ_F in our predictions.
- Our leading order picture $= |\mathcal{M}_2|^2$.

Scale worries

- $W+ \geq n$ jets cross-sections from CDF Run I, compared with (enhanced) leading order theory:

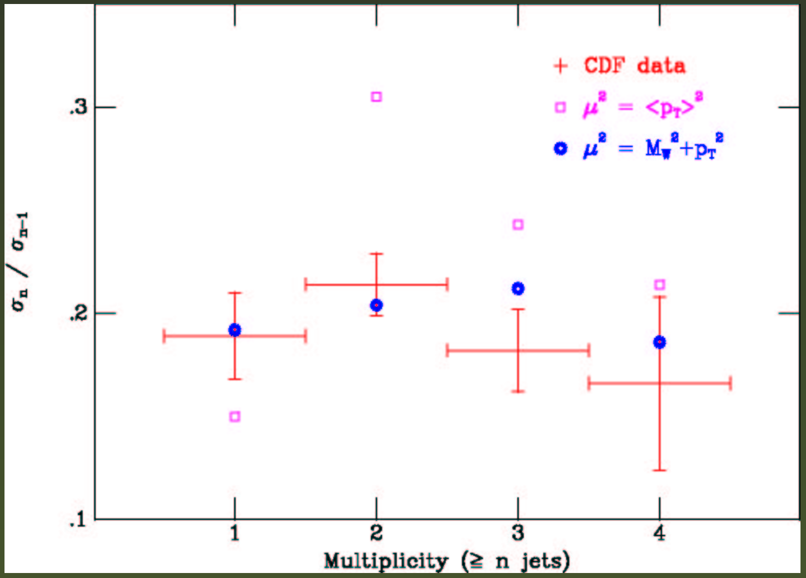


$$\mu_R = \mu_F \equiv \mu$$

- To reproduce the raw cross-sections, especially for the $W + 1, 2$ jet data, the low scale $\mu^2 = \langle p_T \rangle^2$ is preferred.

Scale worries, continued

- Ratio of n-jet cross sections, σ_n/σ_{n-1}:



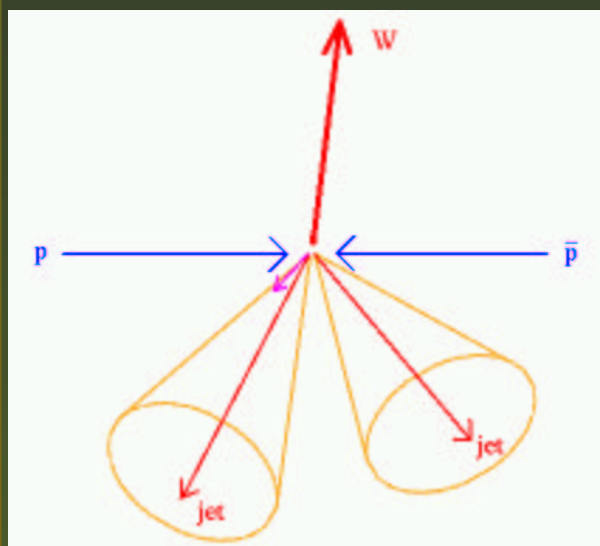
$$\mu_R = \mu_F \equiv \mu$$

- Measures the "reduction in cross section caused by adding a jet" (roughly $\sim \alpha_S$).
- Useful quantity since systematics should cancel.
- High scale $\mu^2 = M_W^2 + p_T^2$ now much closer to data.

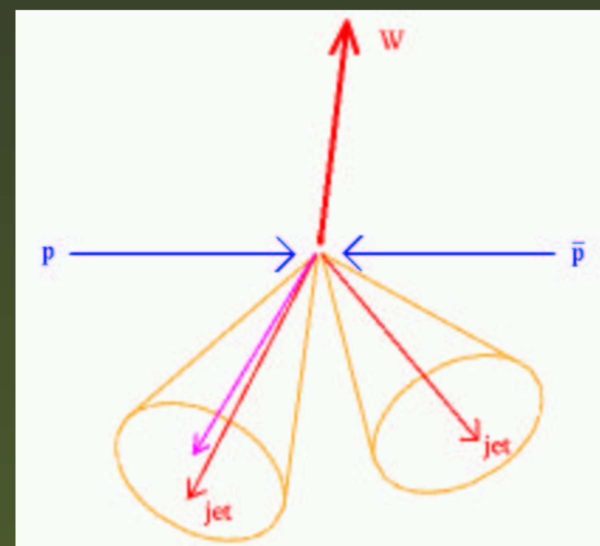
Next-to-leading order

- At next-to-leading order, we include an extra "unresolved" parton in the final state

soft



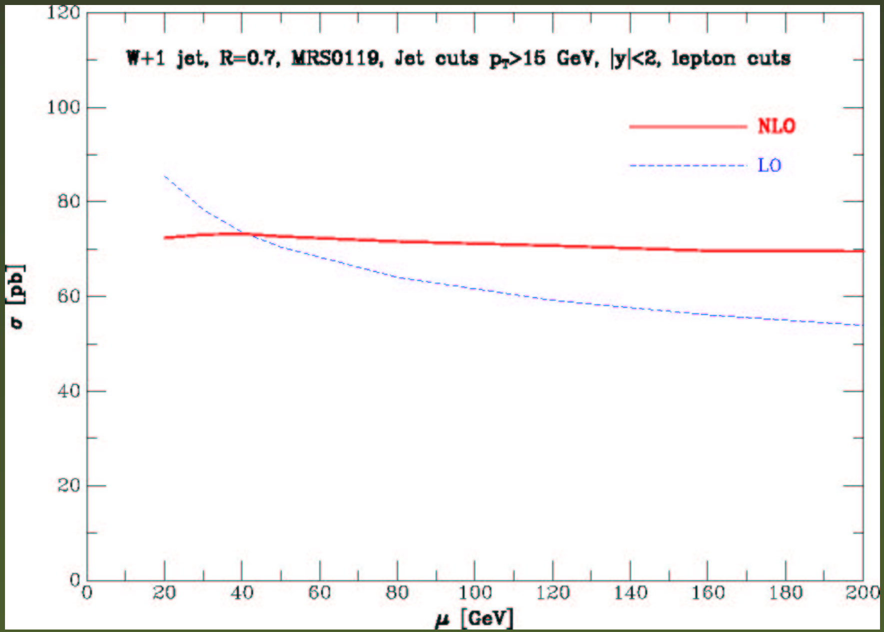
collinear



- The theory begins to look more like an experimental jet, so one expects a better agreement with data.

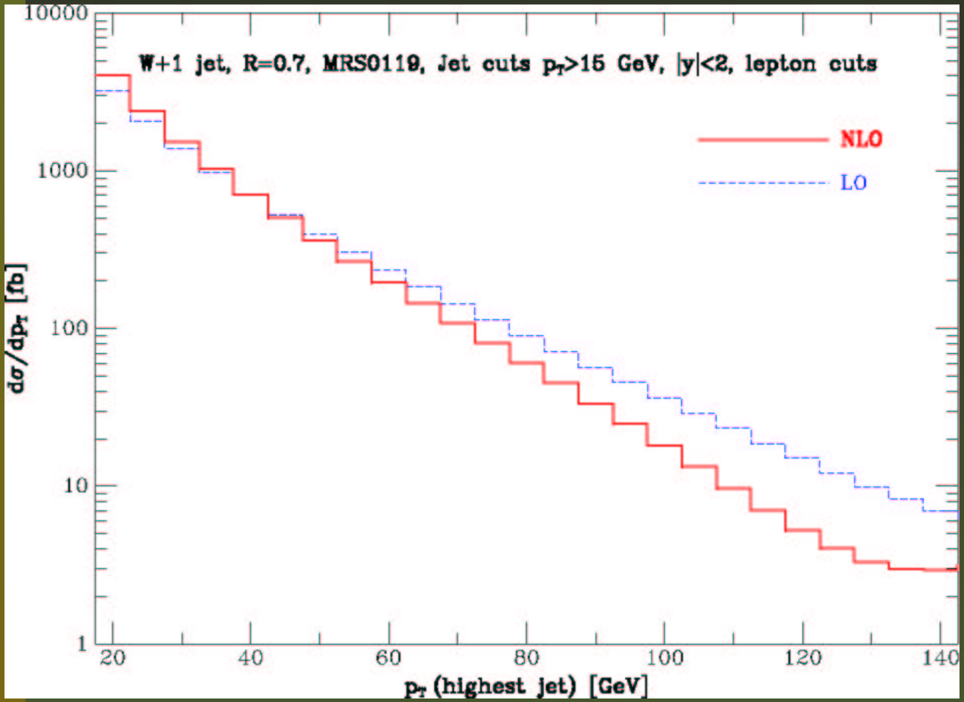
Scale dependence

- $W + 1$ jet cross-section demonstrates the reduced scale dependence that is expected at NLO, as large logarithms are partially cancelled.



- Change between low ~ 20 GeV and high ~ 80 GeV scales is about 30% at LO and $< 5\%$ at NLO.

Jet p_T distribution



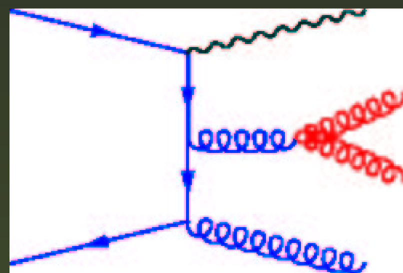
$\mu = 80$ GeV

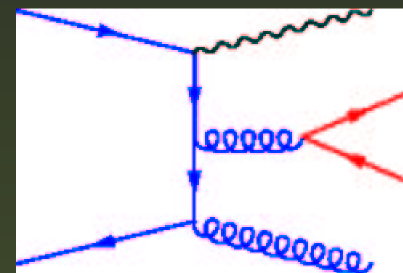
- Leading E_T jet becomes much softer at NLO.
- Depletion in the high-E_T tail because these jets are more likely to radiate a parton that is observed as an extra jet (exclusive sample here, not inclusive).

$W + 2$ jets, NLO theory

- Feynman diagrams for extra parton radiation, e.g.

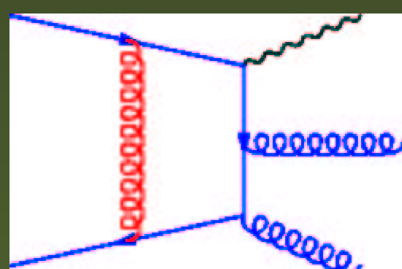
soft gluon

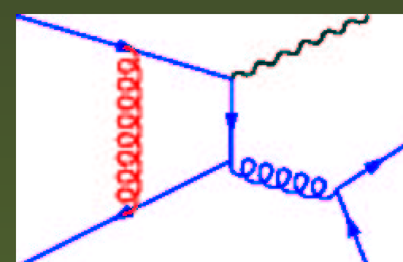




collinear quark

- Loop diagrams, also one extra factor of α_S:





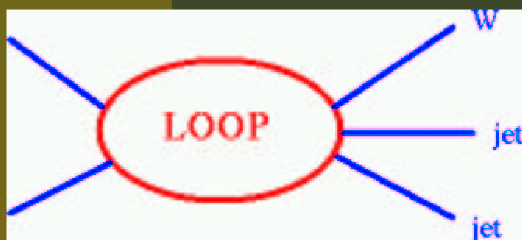
- Results for both exist in the literature, under the guise of $e^+e^- \to 4$ jet matrix elements.

NLO difficulties

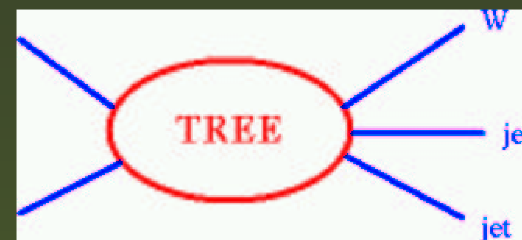
- We must somehow combine two types of diagrams, each with a different number of final state partons.
- Whilst this procedure is well understood from the theory point of view, it does raise problems:
  - There is no simple correspondence between a data event and the theory description.
  - No chance of interfacing with Pythia, since the first stage of the jet evolution is already included (some work in this area at present).
  - Less experimental familiarity with NLO generators.

Loop diagrams

- Use the helicity amplitudes of Z. Bern et al.
- Loop integrals are divergent. The usual choice is to regularize in $d = 4 - 2\epsilon$ dimensions.
- Simplistically, the result is:



$$= \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \times$$

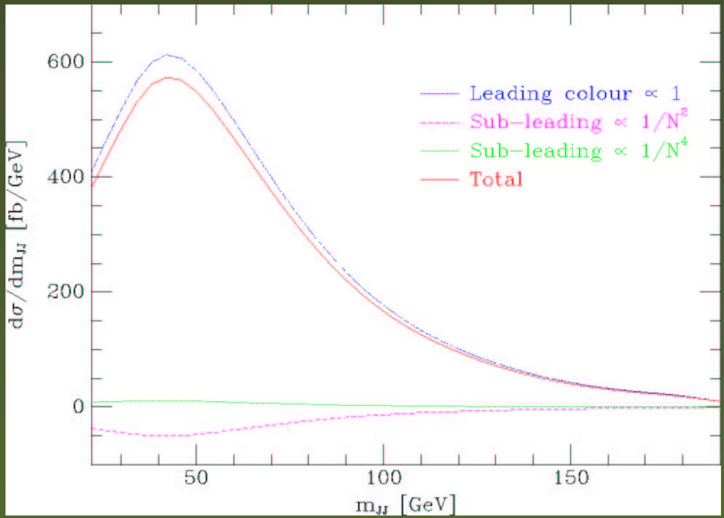


+ finite terms

- The finite terms are rational functions of the invariants, log's and di-log's. There are many terms and they are also slow to evaluate.
- Can improve speed by using the leading colour term.

Colour decomposition

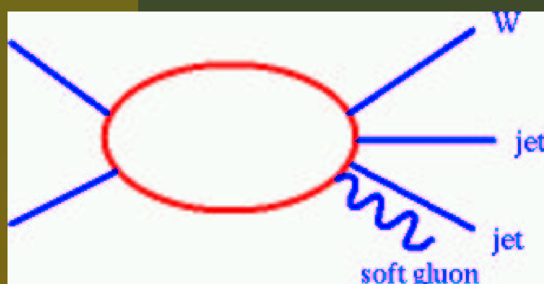
- Recall the two classes of diagrams - ones involving 2 quarks, 2 gluons and those with 4 quarks. We can write the matrix elements for these diagrams as an expansion in the number of colours, N.
- The 2 quark, 2 gluon diagrams contain the leading term and pieces suppressed by $1/N^2$ and $1/N^4$. The 4 quark diagrams are suppressed by $1/N$ and $1/N^3$.



dijet mass distribution

Real diagrams

- The matrix elements for the production of $W + 2$ jets with an extra soft gluon are also divergent, for example in the limit $E_{gluon} \to 0$.
- However, in these diagrams, the matrix elements undergo a remarkable factorization:



$\to$ eikonal $\times$ factor

- The eikonal factor contains all the soft and collinear singularities.
- Exploit this to cancel the singularities.

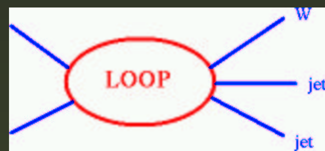
Real diagrams, continued

- Now we must compensate for the singularities that we just cancelled.
- This is done by analytically integrating the eikonal factor over the phase space of the soft gluon, to give:

$$\int (\text{eikonal factor})\, dPS = \frac{D}{\epsilon^2} + \frac{E}{\epsilon} + F$$

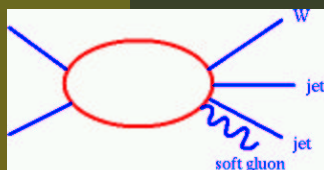
- This is called the subtraction method.
- Careful choice of the kinematics in the lowest-order matrix elements is made, to optimize the singularity cancellation - the dipole subtraction scheme.

Result



$$= \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \times$$

$$\int$$



$$dPS^{\text{gluon}} = \left(\frac{D}{\epsilon^2} + \frac{E}{\epsilon} + F\right) \times$$

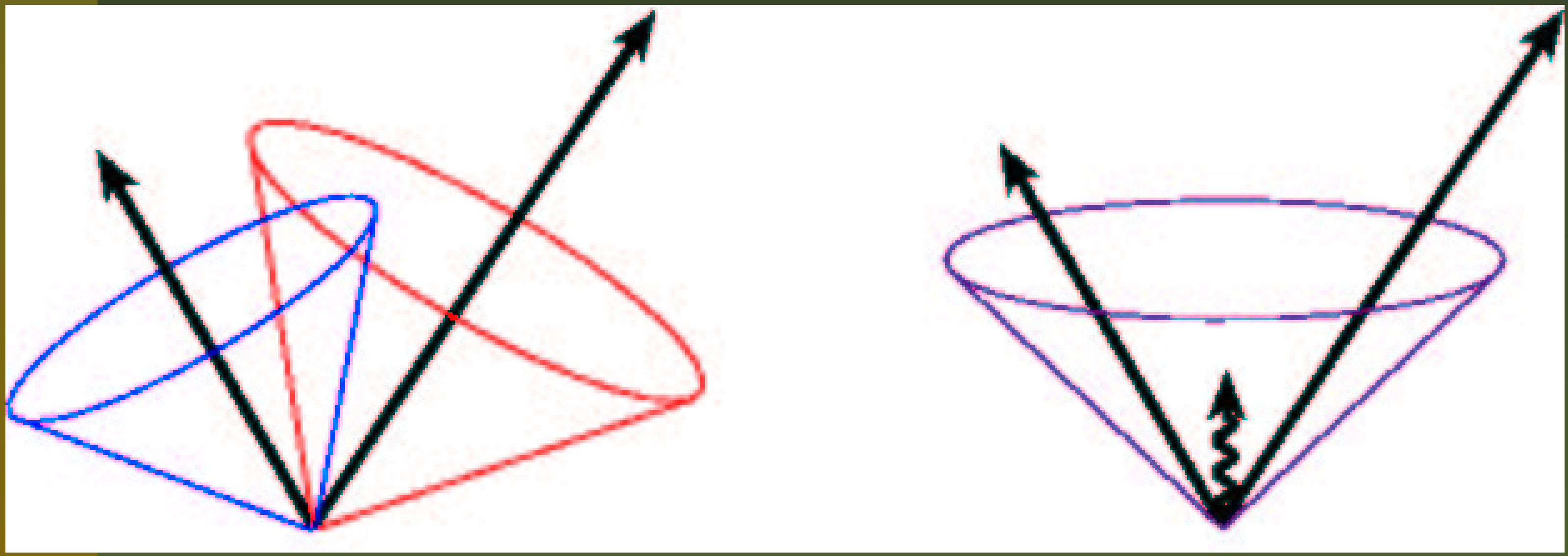
- $A = -D$, $B = -E$ ...so all poles cancel (KLN).
- We are left with integrals over the final 2-jet phase-space for:
  - The remaining finite parts of the loop diagrams;
  - The non-singular real emission diagrams where one jet contains a soft gluon or a collinear quark.

$W + 2$ jet outline

1. Assemble all loop matrix elements - BDKW.
2. Assemble all real radiation matrix elements - NT.
3. Enumerate all possible soft, collinear singularities.
4. Construct appropriate counterterms to cancel these.
5. Check the cancellation occurs in the singular limits.
6. Integrate over the singular areas of phase-space.
7. Check that these poles cancel with those from loops.
8. With a given jet definition and cuts, perform the phase-space integration.
9. Accumulate predictions for any observables required.

Defining a jet - cone algorithm

- Cone-based algorithm, $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} > R$.
- Very popular in Run I.
- Suffers from sensitivity to soft radiation at NLO.



- Instability can be mitigated by extra jet seeds, e.g. midpoint algorithms.

Defining a jet - k_T algorithm

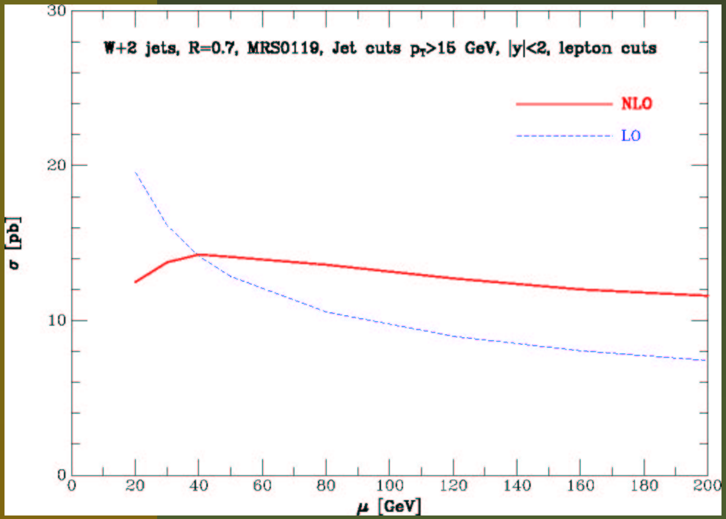
- Preferred by theory - insensitive to soft radiation, immediate matching to resummed calculations.
- Limited experimental use at hadron colliders due to difficulties with energy subtraction.
- Jets are clustered according to the relative transverse momentum of one jet with respect to another.
- Similarity with cone jets is kept, since the algorithm still terminates with all jets having $\Delta R > R$.
- We shall adopt the k_T prescription that is laid out for Run II (G. Blazey et al.), where other ambiguities such as the jet recombination scheme are fixed.

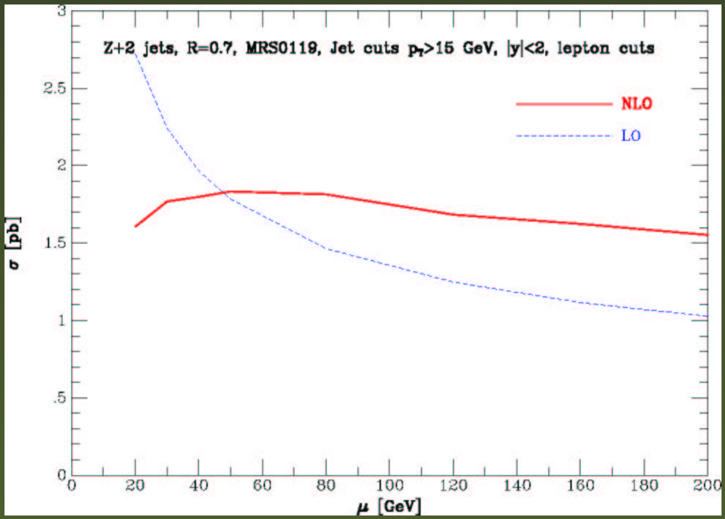
Event cuts

- Concentrate on Tevatron only, cuts chosen accordingly.
- k_T clustering algorithm with pseudo-cone size, $R = 0.7$.
- Jet cuts, $p_T^{\rm jet} > 15$ GeV, $|y^{\rm jet}| < 2$.
- Exclusive (only 2) jets (mostly).
- Lepton cuts, $p_T^{\rm lepton} > 20$ GeV, $|y^{\rm lepton}| < 1$.
- (W only) Missing transverse momentum, $p_T^{\rm miss} > 20$ GeV.
- (Z only) Dilepton mass, $m_{e^- e^+} > 15$ GeV.

Scale dependence

- Choose factorization and renormalization scales to be equal.
- Examine scale dependence of the cross-section integrated over 20 GeV $< m_{JJ} <$ 200 GeV.

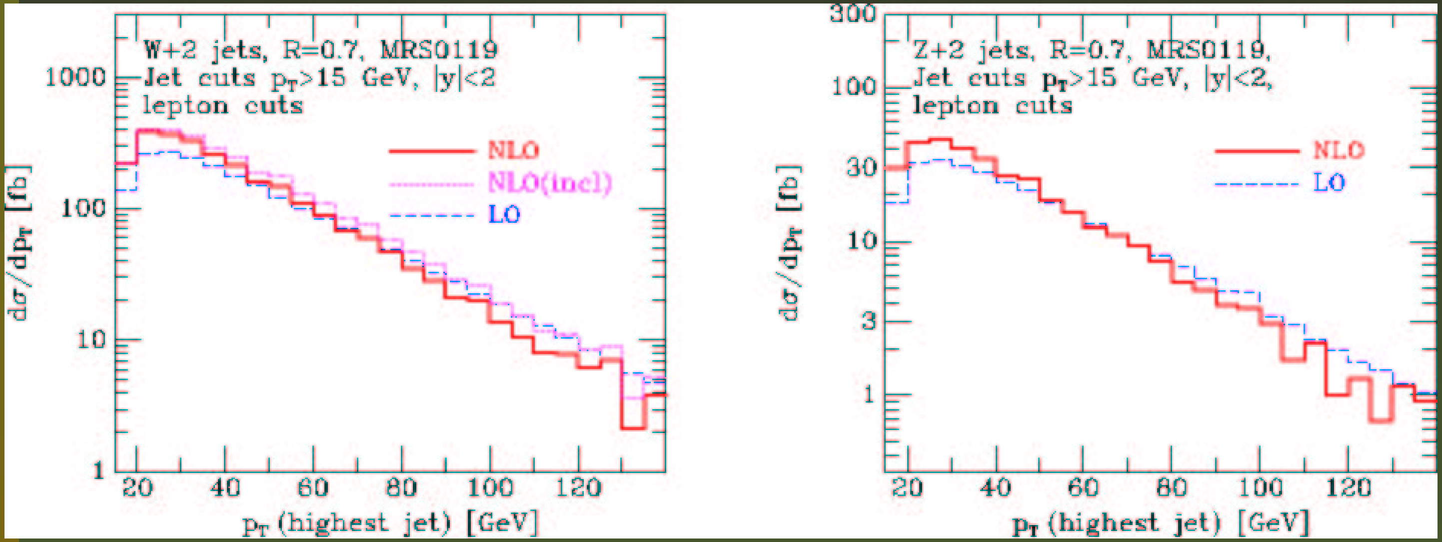




- Scale dependence much reduced from $\sim 100\%$ to $\sim 10\%$ in both cases.

Leading p_T distribution

- p_T distribution of the hardest jet in W, Z+2 jet events, at the scale $\mu = 80$ GeV.



- Turn-over at low p_T since $15\text{ GeV} < p_T^2 < p_T^1$.
- The exclusive spectrum is much softer at next-to-leading order, as in the 1-jet case.
- High-E_T tail is ‘filled in’ for the inclusive case.

Heavy flavour content of $W/Z + 2$ jets

- Many signals of new physics involve the production of a W or Z boson in association with a heavy particle that predominantly decays into a $b\bar{b}$ pair.
- A light Higgs is a prime example and will provide a promising search channel in Run II.

$$p\bar{p} \longrightarrow W(\to e\nu)H(\to b\bar{b})$$

$$p\bar{p} \longrightarrow Z(\to \nu\bar{\nu}, \ell\bar{\ell})H(\to b\bar{b})$$

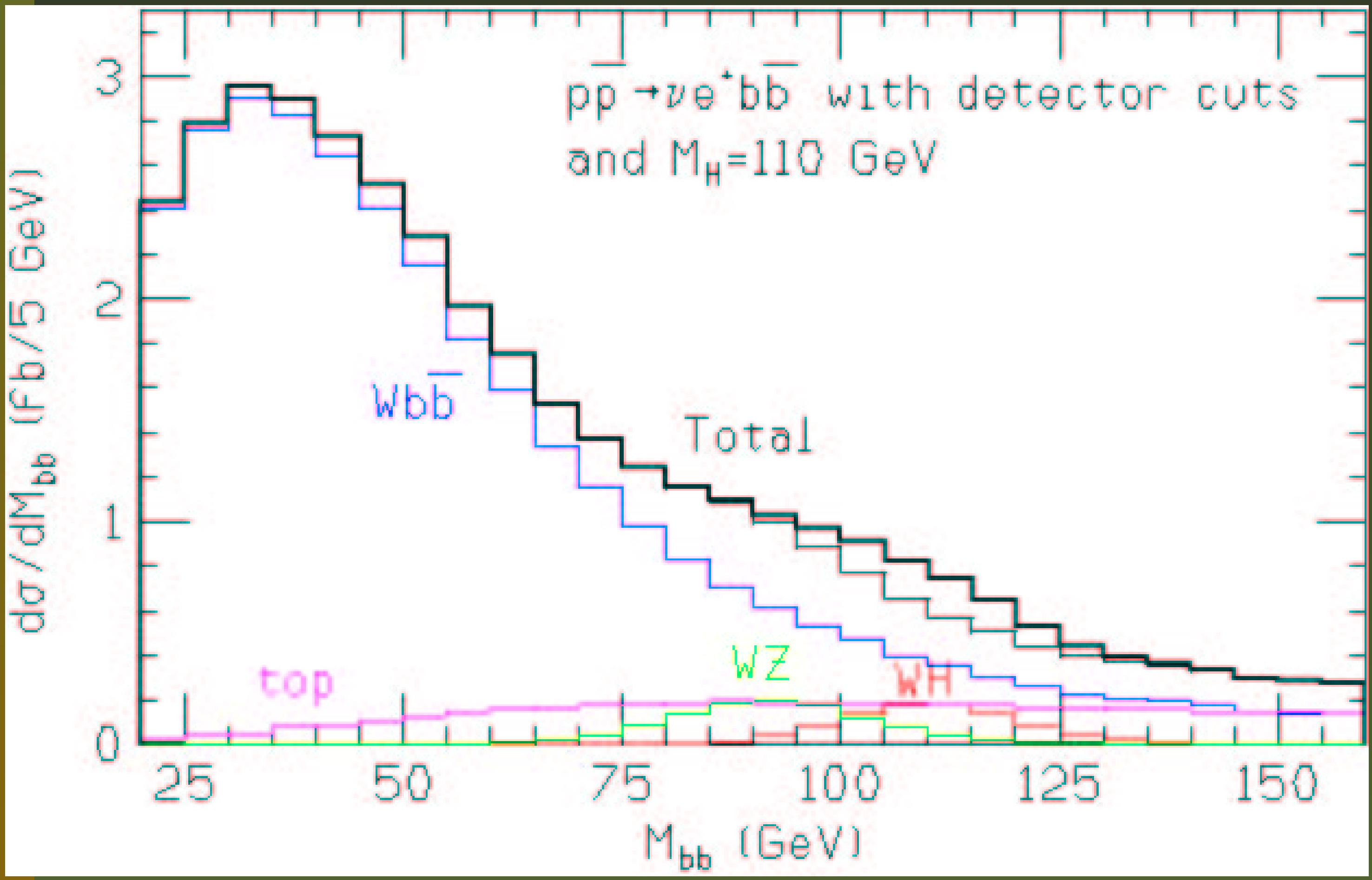
- However, we will need to understand our SM backgrounds very well to perform this search.
- The largest background is 'direct' production:

$$p\bar{p} \longrightarrow W\, g^\star(\to b\bar{b})$$

$$p\bar{p} \longrightarrow Z\, b\bar{b}$$

Background importance

- NLO study of WH search using MCFM.



MCFM Summary - v. 3.0

$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm/Z + H$	$p\bar{p} \to W^\pm/Z + 1\text{ jet}$
$p\bar{p} \to W^\pm + g^\star\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W + 2\text{ jets}$	$p\bar{p} \to Z + 2\text{ jets}$

- MCFM aims to provide a unified description of a number of processes at NLO accuracy.
- Various leptonic and/or hadronic decays of the bosons are included as further sub-processes.
- MCFM version 2.0 is now part of the CDF code repository. Working with experimenters to produce user-friendly input and output, e.g. event ntuples.

Predicting the $Wb\bar{b}$ background

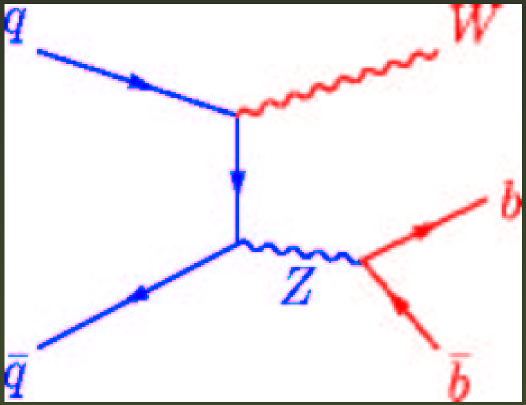
- There are a number of methods for predicting the Standard Model 'direct' background.
- Amongst the theoretical choices are:
  - Fixed order vs. event generator;
  - LO vs. NLO;
  - Pythia vs. Herwig;
  - Massive b's vs. Massless b's.
- Citing a 40% uncertainty on the leading-order calculation (M. Mangano), a recent study by CDF uses a mixed approach relying heavily on generic $W+$ jet data, but with some theoretical input.

Hybrid recipe

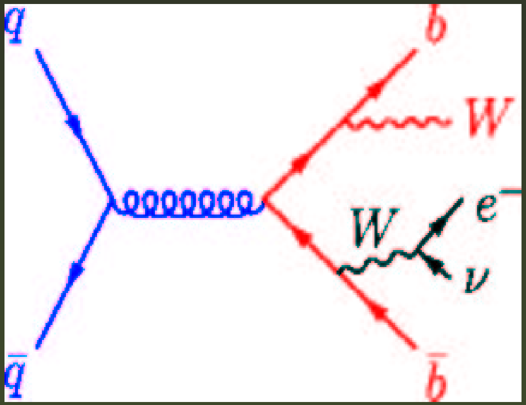
1. Measure the number of $W + 2$ jet events.
2. Subtract the number of events predicted by theory from non-direct channels.
   - $t\bar{t}$ (Pythia norm. to NLO)
   - Diboson (Pythia norm. to NLO)
   - Single top (Pythia/Herwig norm. to NLO)
3. This estimates the number of direct $W + 2$ jet events.
4. Use VECBOS (leading order) + Herwig to estimate the fraction of $W + 2$ jet events that contain two b's.
5. Obtain prediction for direct $W + b\bar{b}$ events.

Other $Wb\bar{b}$ backgrounds

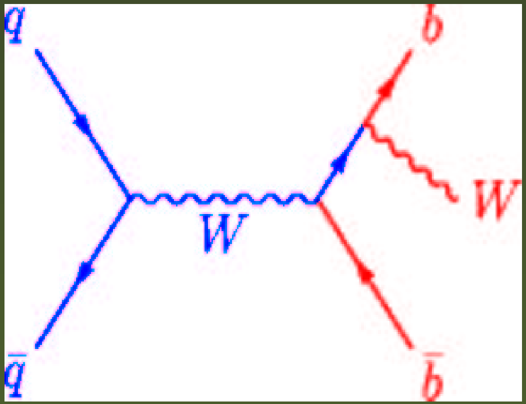
diboson



$t\bar{t}$



single top (s)



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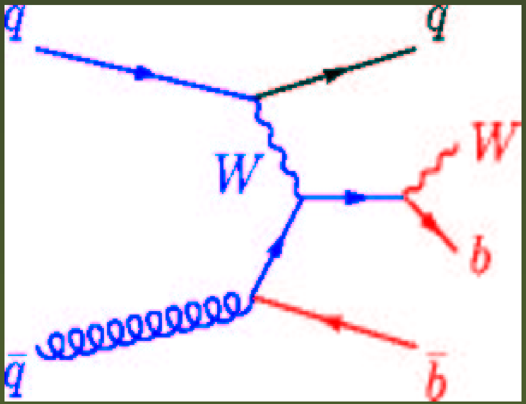


Alternatives

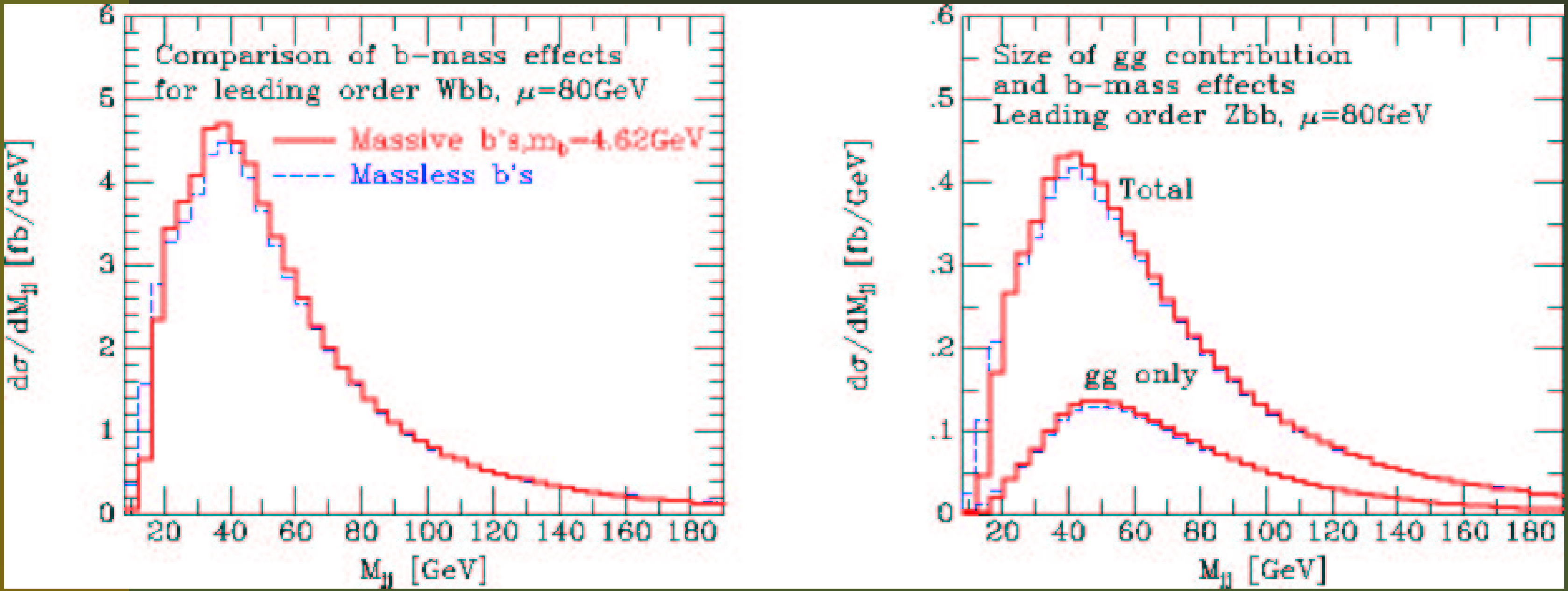
- Is this the best we can do?
- VECBOS suffers from the same leading order uncertainty that we were trying to avoid.
- Herwig can be somewhat of a black box.
- We can calculate the $Wb\bar{b}$ cross-section at NLO in MCFM. This has a much reduced scale dependence, but suffers from no showering and massless b's.
- Another option is to calculate the same fraction,

$$\frac{\sigma(Wb\bar{b})}{\sigma(W+2\,\mathrm{jet})}$$

that is calculated by Herwig, but at NLO. Some systematics (showering, perhaps) should cancel.

b-mass effects

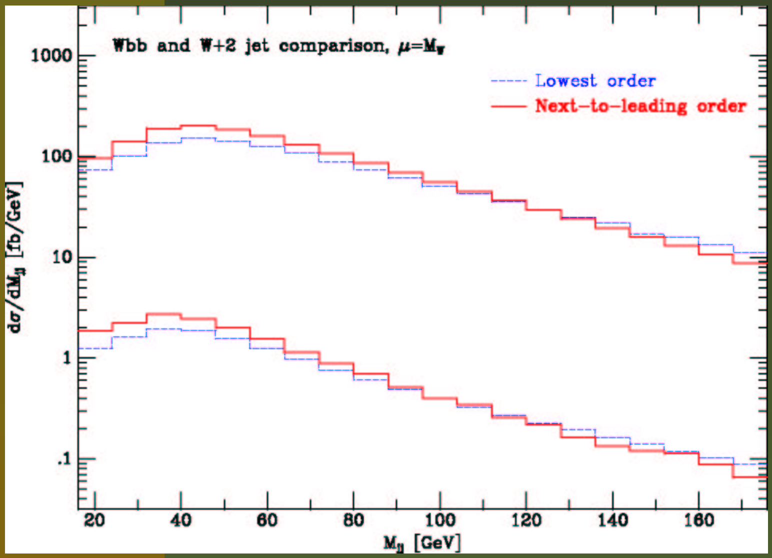
- Compare the lowest order predictions for m_b zero and non-zero.

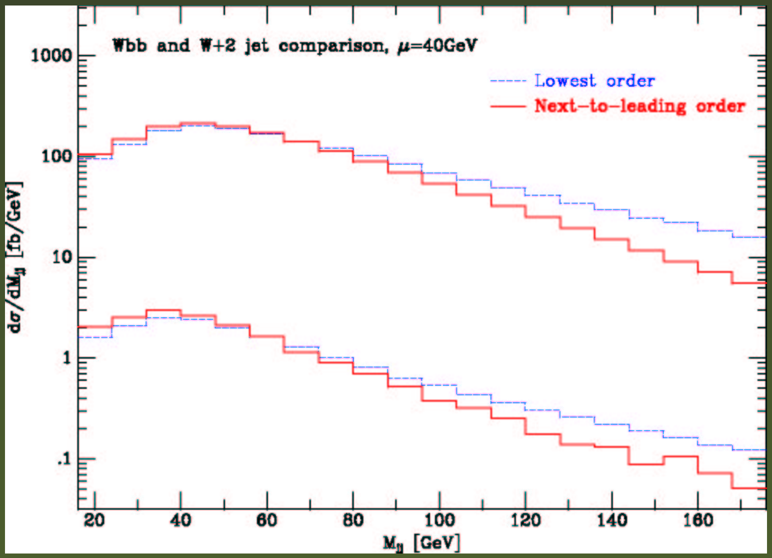


- In the interesting region - the peak at low mass - matrix element effects dominate over phase space. The corrections there are of order 5%.

m_{JJ} distributions

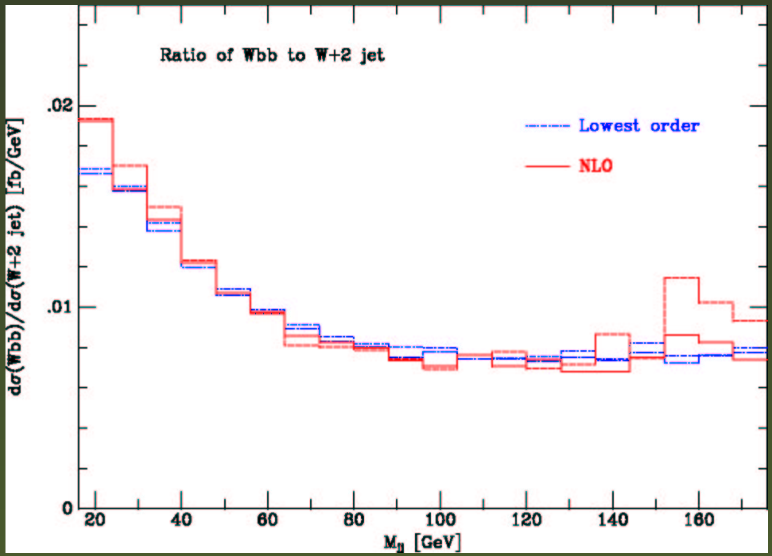
- $Wb\bar{b}$ and $W+2$ jet distributions appear very similar in shape at both LO and NLO. The shapes change when moving to a lower scale, with a depletion in the cross-section at high M_{jj}.





Heavy flavour fraction

- The ratio of b-tagged to untagged jets changes very little at NLO and appears to be predicted very well by perturbation theory.



- The fraction is peaked at low M_{jj}, where it is approximately 2.5 times as high as the fairly constant value of 0.8% for $M_{jj} > 60$ GeV.

Conclusions

- We have presented the first results for $W, Z + 2$ jet production at next-to-leading order.
- Scale dependence is greatly reduced to $\sim 10\%$ and distributions are considerably changed upon including QCD corrections.
- The fraction of a $W + 2$ jet sample that contains two b-jets is predicted very well in perturbation theory.
- Results for the LHC are forthcoming.
- There are many interesting studies to be done - from tests of QCD to backgrounds for new physics.